

Article

Input Noise Variance Estimation for Adaptive Filtering Without Prior Information

Jae Jin Jeong 

School of Electrical Engineering, Kumoh National Institute of Technology, Gumi-si 39177, Republic of Korea; jeong@kumoh.ac.kr

Abstract

In adaptive filtering, input noise degrades the performance of standard algorithms by introducing bias into the weight estimate. This paper presents a bias-compensated adaptive filtering method that estimates the input noise variance without requiring any prior information. By exploiting the orthogonality between the estimated weight vector and the error vector, the proposed Prior-Free Noise Variance Estimator (PFNVE) obtains a reliable noise variance estimate that is independent of the output noise characteristics and the input-to-output noise ratio. System identification experiments show that the PFNVE achieves steady-state accuracy comparable to existing techniques while offering noticeably faster tracking when the system undergoes abrupt changes. Since the PFNVE does not require any structural modification to the underlying filter, it can be directly applied to other adaptive filters, including MS-PNLMS. Performance is evaluated through simulations on dense and sparse systems under various conditions, including colored input and time-varying noise.

Keywords: bias-compensated NLMS; bias-compensated MS-PNLMS; noisy input; system identification

1. Introduction

Adaptive filters have found widespread use in channel estimation, equalization, beamforming, active noise control, and inverse modeling [1–4]. The Normalized Least Mean Square (NLMS) algorithm, in particular, has been widely adopted owing to its simplicity and stable convergence behavior; it continues to appear in recent work, for example, in fault diagnosis applications [5], power systems [6,7], and beamforming control [8]. It is well known, however, that the performance of NLMS deteriorates when the input signal is contaminated by noise [9], since the noise introduces a bias into the weight estimate that slows convergence and increases the steady-state error [10].

This bias has been recognized as a fundamental challenge since the early development of adaptive filtering [11,12]. While the steady-state error caused by output noise decreases as the step size is reduced, the bias induced by input noise distorts the input correlation matrix and persists regardless of the step size [13]. Indeed, such input noise is frequently encountered in a variety of practical signal processing systems. In acoustic echo cancellation, the far-end signal picked up by a microphone is inevitably corrupted by background noise before entering the adaptive filter [14–16]. A similar problem appears in wireless channel estimation, where thermal noise and interference corrupt the received pilot signal, so that the adaptive filter observes a noisy copy of the true transmitted sequence [10,17]. Active noise control (ANC) systems face the same difficulty: the reference microphone



Academic Editor: Sergio Toscani

Received: 20 March 2026

Revised: 3 April 2026

Accepted: 10 April 2026

Published: 13 April 2026

Copyright: © 2026 by the author.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

captures not only the target noise source but also sensor self-noise and secondary-path disturbances [18,19]. Furthermore, in modern applications such as vehicle interior noise reduction and industrial vibration control, standard Filtered- x NLMS (FxNLMS) algorithms often suffer from significantly degraded noise attenuation when reference signals are contaminated [20,21]. Under all of these conditions, the standard NLMS algorithm yields biased weight estimates, and the resulting performance degradation can be substantial [22]. Therefore, various robust NLMS variants have been proposed to mitigate the effects of such noise [23].

To specifically tackle the weight bias induced by input noise, several bias-compensation strategies have been developed. The C-NLMS algorithm [24] achieves consistency by correcting the noisy input correlation matrix; however, its noise variance estimate ignores output noise entirely, limiting its accuracy to scenarios where the output noise power is much smaller than the input noise power. BC-NLMS [25] improves on this by incorporating the input-to-output noise variance ratio ρ , which extends its applicability to a broader range of SNR conditions. Alternatively, BC-NSAF [26] replaces the noise ratio with a shrinkage threshold derived from the output noise variance σ_v^2 . These approaches suppress weight bias and perform well under static conditions.

Driven by these successes, many recent studies have extended bias compensation techniques to a wider variety of adaptive filtering structures. Zhao et al. [27] designed a sign subband adaptive filter with per-tap weighting factors for input noise, thereby gaining robustness against sign-based quantization errors. Song et al. [28] proposed a robust bias-compensated subband adaptive filter capable of handling both noisy inputs and impulsive interference. A bias-compensated sign algorithm with optimized step size was also developed for noisy input environments [29]. In a related effort, Zhao et al. [30,31] combined the maximum correntropy criterion with subband filtering to deal simultaneously with impulsive noise and input noise. Park et al. [32] brought bias compensation into the affine projection framework together with a variable step-size mechanism, and Gu et al. [33] developed a constrained adaptive filtering algorithm that performs bias compensation under linear constraints.

However, all the aforementioned methods share a critical limitation: they strictly rely on prior noise information, such as the noise variance ratio, the exact output noise power, or a related threshold. This requirement limits their applicability in time-varying environments. In practical implementations, continuously updating these prior parameters requires additional sensors or periodic halting of the system for recalibration, imposing structural burdens. For example, in an active noise control (ANC) system for vehicle interiors or portable headphones, the primary noise characteristics and background interference can fluctuate rapidly as the user moves through different acoustic environments [34]. If the bias compensation relies on a fixed noise ratio or a one-time calibration of the output noise power, the filter will operate with stale parameters after such a change, leading to degraded tracking and elevated steady-state error. This motivates the development of an estimation method that continuously adapts to the current noise conditions without any externally supplied parameters.

In this paper, we propose the Prior-Free Noise Variance Estimator (PFNVE) that requires no prior information. The key idea is to exploit the orthogonality between the weight-error vector and the current weight vector during the update process, which approximately removes the bias induced by the corrupted input correlation matrix. This yields an unbiased variance estimate in real time, independent of the output noise level or the input-to-output noise variance ratio. To verify the performance of the proposed method, extensive system identification experiments were conducted under dense and sparse environments across various signal-to-noise ratio (SNR) conditions. The simulation

results show that the resulting filter matches the steady-state accuracy of BC-NLMS and BC-NSAF while offering faster tracking when the system undergoes abrupt parameter changes. Furthermore, because the estimator operates solely on internal variables already available in any standard LMS-type update, it transfers to other adaptive filter structures without algorithmic modification. We verify this applicability by integrating the proposed method with the recently studied MS-PNLMS algorithm [35], showing that comparable performance can be achieved at similar computational cost.

The remainder of this paper is organized as follows. Section 2 reviews the conventional bias-compensated algorithms. Section 3 derives the proposed noise variance estimator and discusses its properties. Section 4 presents the simulation results under various conditions including time-varying noise and colored input. Finally, conclusions are drawn in Section 5.

2. Review of the Conventional Algorithm

The desired signal $d(n)$ is defined as:

$$d(n) = \mathbf{u}_n^T \mathbf{w}_o + v(n), \quad (1)$$

where $\mathbf{u}_n = [u(n), u(n-1), \dots, u(n-M+1)]^T$ denotes the noise-free input vector; $u(n)$ is the input signal at time index n , that is a white Gaussian sequence of zero-mean with σ_u^2 ; $\mathbf{w}_o \in \mathcal{R}^{M \times 1}$ is the optimal weight vector; M means the tap length of the unknown system; $(\cdot)^T$ denotes the transpose of the vector or matrix; $v(n)$ is the measurement noise, and it is a zero-mean white Gaussian noise with σ_v^2 .

The bias-compensated NLMS and MS-PNLMS algorithms employ a realistic model for the filter input signal by considering it as a noisy version, expressed as:

$$\bar{\mathbf{u}}_n = \mathbf{u}_n + \boldsymbol{\eta}_n, \quad (2)$$

where $\boldsymbol{\eta}_n = [\eta(n), \eta(n-1), \dots, \eta(n-M+1)]^T$ denotes the input noise vector, modeled as a zero-mean white Gaussian process with variance σ_η^2 .

2.1. Review of Bias-Compensated NLMS (BC-NLMS) Algorithm

The update equation of BC-NLMS algorithm is given by [25]:

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \frac{\bar{\mathbf{u}}_n \bar{e}(n) + \hat{\sigma}_\eta^2(n) \mathbf{w}_n}{\bar{\mathbf{u}}_n^T \bar{\mathbf{u}}_n}, \quad (3)$$

where μ is the step size, $\bar{e}(n)$ is the error signal defined as $\bar{e}(n) = d(n) - \bar{\mathbf{u}}_n^T \mathbf{w}_n$, and $\hat{\sigma}_\eta^2(n)$ represents an estimate of the input noise variance.

Several approaches have been proposed for estimating the input noise variance. The first method, which utilizes the ratio between input and output noise variances, uses the following formula [25]:

$$\hat{\sigma}_\eta^2(n) = \frac{\hat{\sigma}_e^2(n)}{M \hat{\sigma}_w^2(n) + \rho}, \quad (4)$$

where ρ is the input-output noise variance ratio and

$$\hat{\sigma}_e^2(n) = \alpha \hat{\sigma}_e^2(n-1) + (1-\alpha) \bar{e}^2(n), \quad (5)$$

$$\hat{\sigma}_w^2(n) = \alpha \hat{\sigma}_w^2(n-1) + (1-\alpha) \frac{\mathbf{w}_n^T \mathbf{w}_n}{M}, \quad (6)$$

with α being a smoothing parameter.

Another approach employs the shrinkage denoising method and is formulated as [26]:

$$\hat{\sigma}_{\eta}^2(n) = \frac{\hat{\sigma}_{\varepsilon}^2(n)}{M\hat{\sigma}_w^2(n)}, \quad (7)$$

where

$$\hat{e}(n) = \text{sign}(\bar{e}(n)) \max(|\bar{e}(n)| - \tau, 0), \quad (8)$$

$$\hat{\sigma}_{\varepsilon}^2(n) = \alpha\hat{\sigma}_{\varepsilon}^2(n-1) + (1-\alpha)\hat{e}^2(n), \quad (9)$$

$$\hat{\sigma}_w^2(n) = \alpha\hat{\sigma}_w^2(n-1) + (1-\alpha)\frac{\mathbf{w}_n^T \mathbf{w}_n}{M}, \quad (10)$$

here, $\text{sign}(\cdot)$ denotes the sign function and $\tau = \sqrt{\kappa\sigma_v^2}$.

However, both methods are limited by their reliance on prior knowledge of either the input-output noise variance ratio or the exact value of the output noise variance. This requirement can significantly restrict their applicability in practical scenarios where such prior information is unavailable or difficult to obtain.

2.2. Review of Bias-Compensated MS-PNLMS Algorithm

The update equation of the Bias-compensated MS-PNLMS (BC-MS-PNLMS) algorithm is presented as [35]

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \frac{\mathbf{G}_n \bar{\mathbf{u}}_n \bar{e}(n) + \hat{\sigma}_{\eta}^2(n) \mathbf{G}_n \mathbf{w}_n}{\bar{\mathbf{u}}_n^T \mathbf{G}_n \bar{\mathbf{u}}_n}, \quad (11)$$

$$\mathbf{G}_n = \text{diag}\{g_0(n), g_1(n), \dots, g_{M-1}(n)\}, \quad (12)$$

where $\mathbf{G}_n$ denotes the step-size matrix for the individual taps of the filter. The diagonal elements of $\mathbf{G}_n$ are calculated as

$$f(x) = \begin{cases} \frac{x}{b} & x < b \\ \frac{x+b}{cb} + a - 2 & (c-1)b < x < (2c-1)b \end{cases}, \quad (13)$$

$$\beta(n) = \max\{\delta, f(w_0(n)), f(w_1(n)), \dots, f(w_{M-1}(n))\}, \quad (14)$$

$$g_m(n) = \max\{\gamma\beta(n), f(w_m(n))\}, \quad (15)$$

where $c = 2^{a-1}$, $\max\{a, b\}$ is the maximum value between a and b , $m = 0, 1, \dots, M-1$, and γ means the user parameter.

The BC-MS-PNLMS algorithm estimates input noise variance using the same technique as BC-NLMS [35].

$$\hat{\sigma}_{\eta}^2(n) = \frac{1}{\frac{M\hat{\sigma}_w^2(n)+\rho}{\hat{\sigma}_{\varepsilon}^2(n)} + \frac{M}{\bar{\mathbf{u}}_n^T \bar{\mathbf{u}}_n}}, \quad (16)$$

where ρ is the input-output noise variance ratio, and

$$\hat{\sigma}_{\varepsilon}^2(n) = \alpha\hat{\sigma}_{\varepsilon}^2(n-1) + (1-\alpha)\bar{e}^2(n), \quad (17)$$

$$\hat{\sigma}_w^2(n) = \alpha\hat{\sigma}_w^2(n-1) + (1-\alpha)\frac{\mathbf{w}_n^T \mathbf{w}_n}{M}, \quad (18)$$

with α being a smoothing parameter.

As shown in the above equations, the BC-MS-PNLMS algorithm also depends on the input-to-output noise variance ratio ρ . Consequently, the conventional bias-compensated methods reviewed in this section share a common limitation: they require prior noise information.

3. Prior-Free Bias-Compensated NLMS

This section derives a noise variance estimator that does not require any prior information. Specifically, the derivation begins by forming the product of the filter output and the error signal, expressed as follows:

$$y(n)\bar{e}(n) = \mathbf{w}_n^T \bar{\mathbf{u}}_n (\bar{\mathbf{u}}_n^T \tilde{\mathbf{w}}_n + v(n) - \boldsymbol{\eta}_n^T \mathbf{w}_o) \quad (19)$$

Applying the orthogonality principle, which implies $E\{\tilde{\mathbf{w}}_n \mathbf{w}_n\} \approx 0$ [25], and assuming that (i) the input noise vector $\boldsymbol{\eta}_n$ is independent of $\mathbf{w}_n$, (ii) the measurement noise $v(n)$ is zero-mean and independent of $\mathbf{w}_n$ and $\bar{\mathbf{u}}_n$, and (iii) $\boldsymbol{\eta}_n$ is independent of the clean input component, the expectation of the above expression yields

$$\begin{aligned} E\{\mathbf{w}_n^T \bar{\mathbf{u}}_n (\bar{\mathbf{u}}_n^T \tilde{\mathbf{w}}_n + v(n) - \boldsymbol{\eta}_n^T \mathbf{w}_o)\} &= \text{Tr}\left(E\{\bar{\mathbf{u}}_n \bar{\mathbf{u}}_n^T \tilde{\mathbf{w}}_n \mathbf{w}_n^T\}\right) - \text{Tr}\left(E\{\bar{\mathbf{u}}_n \boldsymbol{\eta}_n^T (\tilde{\mathbf{w}}_n + \mathbf{w}_n) \mathbf{w}_n^T\}\right) \\ &= \text{Tr}\left(E\{\bar{\mathbf{u}}_n \bar{\mathbf{u}}_n^T \tilde{\mathbf{w}}_n \mathbf{w}_n^T\}\right) - \text{Tr}\left(E\{\bar{\mathbf{u}}_n \boldsymbol{\eta}_n^T \tilde{\mathbf{w}}_n \mathbf{w}_n^T\}\right) \\ &\quad - \text{Tr}\left(E\{\bar{\mathbf{u}}_n \boldsymbol{\eta}_n^T \mathbf{w}_n \mathbf{w}_n^T\}\right) \\ &= -\text{Tr}\left(E\{\boldsymbol{\eta}_n \boldsymbol{\eta}_n^T \mathbf{w}_n \mathbf{w}_n^T\}\right) \\ &= -\sigma_{\boldsymbol{\eta}}^2 \sigma_w^2. \end{aligned} \quad (20)$$

Rearranging the above in terms of $\sigma_{\boldsymbol{\eta}}^2(n)$, the following expression is obtained:

$$\hat{\sigma}_{\boldsymbol{\eta}}^2(n) = \frac{|\gamma(n)|}{\hat{\sigma}_w^2(n) + \epsilon}, \quad (21)$$

where ϵ is a small regularization parameter. The terms $\gamma(n)$ and $\hat{\sigma}_w^2(n)$ are estimated as

$$\gamma(n) = \alpha \gamma(n-1) + (1-\alpha) y(n) \bar{e}(n), \quad (22)$$

$$\hat{\sigma}_w^2(n) = \alpha \hat{\sigma}_w^2(n-1) + (1-\alpha) \mathbf{w}_n^T \mathbf{w}_n. \quad (23)$$

The absolute value of $\gamma(n)$ is taken to ensure $\hat{\sigma}_{\boldsymbol{\eta}}^2(n)$ remains non-negative, as $\gamma(n)$ itself can take negative values. Furthermore, to reduce variability caused by noise and outliers during the estimation process of $\gamma(n)$, a median filter is used [36]. Specifically, $\gamma(n)$ is updated as follows:

$$\gamma(n) = \alpha \gamma(n-1) + (1-\alpha) \text{med}(A(n)), \quad (24)$$

where $\text{med}(\cdot)$ denotes the median operator over a sliding window of length N_w and $A(n) = [y(n)\bar{e}(n), y(n-1)\bar{e}(n-1), \dots, y(n-N_w+1)\bar{e}(n-N_w+1)]$.

Remark 1. The orthogonality relation $E\{\tilde{\mathbf{w}}_n \mathbf{w}_n\} \approx 0$ holds in the vicinity of convergence [25]. Near the steady state, $\tilde{\mathbf{w}}_n$ is dominated by random gradient noise and fluctuates around zero, rendering it statistically orthogonal to $\mathbf{w}_n$. During the initial transient phase or following an abrupt system change, however, $\tilde{\mathbf{w}}_n$ is large and this relation does not hold exactly. To mitigate this, the exponential smoother with factor α attenuates rapid fluctuations in the estimate, while the median filter over N_w samples suppresses outlier products $y(n)\bar{e}(n)$ that arise when the weight error is temporarily large. Furthermore, due to the finite window size N_w and large transient fluctuations, the statistical cancellation of cross-terms in $y(n)\bar{e}(n)$ may be incomplete, occasionally yielding a negative variance estimate. The absolute-value operation is therefore applied to prevent a negative $\hat{\sigma}_{\boldsymbol{\eta}}^2(n)$, which would otherwise destabilize the update in (3). Once the filter returns to the neighborhood of the optimum, the orthogonality relation is restored and the estimator resumes reliable variance tracking.

The proposed bias-compensated adaptive filtering method is summarized in Algorithm 1. Although the weight update step is written based on the standard BC-NLMS formulation, the same algorithm can be applied by substituting other LMS-family update equations, such as MS-PNLMS, without altering the noise variance estimation process.

Algorithm 1 BC-NLMS with PFNVE

```

1: Initialization:
2:    $\mathbf{w}_0 = \mathbf{0}_{M \times 1}$ ,  $\gamma(-1) = 0$ ,  $\hat{\sigma}_w^2(-1) = 0$ 
3: Parameters:
4:   Step size  $\mu$ , smoothing factor  $\alpha$ , median filter window length  $N_w$ , regularization  $\epsilon$ 
5: Computation:
6: for  $n = 0, 1, 2, \dots$  do
7:   Get the input vector  $\bar{\mathbf{u}}_n$  and desired signal  $d(n)$ 
8:   Calculate the filter output:
9:      $y(n) = \bar{\mathbf{u}}_n^T \mathbf{w}_n$ 
10:  Calculate the error signal:
11:     $\bar{e}(n) = d(n) - y(n)$ 
12:  Construct the sliding window buffer:
13:     $A(n) = [y(n)\bar{e}(n), y(n-1)\bar{e}(n-1), \dots, y(n-N_w+1)\bar{e}(n-N_w+1)]$ 
14:  Update the intermediate estimates:
15:     $\gamma(n) = \alpha\gamma(n-1) + (1-\alpha)\text{med}(A(n))$ 
16:     $\hat{\sigma}_w^2(n) = \alpha\hat{\sigma}_w^2(n-1) + (1-\alpha)\mathbf{w}_n^T \mathbf{w}_n$ 
17:  Estimate the input noise variance:
18:     $\hat{\sigma}_\eta^2(n) = \frac{|\gamma(n)|}{\hat{\sigma}_w^2(n) + \epsilon}$ 
19:  Update the filter weight vector:
20:     $\mathbf{w}_{n+1} = \mathbf{w}_n + \mu \frac{\bar{\mathbf{u}}_n \bar{e}(n) + \hat{\sigma}_\eta^2(n) \mathbf{w}_n}{\bar{\mathbf{u}}_n^T \bar{\mathbf{u}}_n}$ 
21: end for
  
```

Computational complexity is a critical issue in practical applications of adaptive filters; therefore, an analysis of the computational requirements is presented in Table 1. Table 1 compares the computational complexity of the noise variance estimation step in terms of multiplications per iteration. By utilizing the scalar product of the filter output and the error signal, $y(n)\bar{e}(n)$, the proposed method maintains low computational demand. Specifically, it requires $M + 6$ multiplications plus a median selection over N_w elements, which adds negligible overhead for small N_w .

Table 1. Computational complexity analysis of input noise variance estimation methods.

Algorithm	Multiplications per Iteration
BC-NLMS	$M + 8$
BC-NSAF	$M + 9$
BC-MS-PNLMS	$2M + 9$
PFNVE-based BC-NLMS	$M + 6$

4. Simulation Results

This section evaluates the PFNVE-based BC-NLMS algorithm through system identification experiments. The algorithm was compared with NLMS, BC-NLMS, and BC-NSAF methods in dense systems, and with BC-MS-PNLMS in sparse systems. To ensure a fair comparison with these full-band algorithms, the BC-NSAF was implemented with a single

subband ($N = 1$). The adaptive filter length matched that of the unknown system. In each trial, the unknown system was generated as a random vector drawn from a standard normal distribution and normalized to unit norm.

The input signal $u(n)$ was an independent and identically distributed (i.i.d.) Gaussian sequence with unit variance unless otherwise stated. The input noise $\eta(n)$ and output noise $v(n)$ were generated as independent zero-mean white Gaussian processes. The input SNR is defined as $10 \log_{10}(\sigma_u^2 / \sigma_\eta^2)$, and the output SNR is defined as $10 \log_{10}(E\{(\mathbf{u}_n^T \mathbf{w}_o)^2\} / \sigma_v^2)$. All algorithms were initialized with the zero vector. Performance was evaluated using normalized mean square deviation (NMSD) in decibels, defined as $10 \log_{10}(\|\mathbf{w}_n - \mathbf{w}_o\|^2 / \|\mathbf{w}_o\|^2)$, averaged over 100 independent trials.

4.1. Dense System

Performance of the PFNVE-based BC-NLMS algorithm in dense systems is presented in comparison with NLMS, BC-NLMS, and BC-NSAF. All algorithms used a step size of 0.2 for fairness. Parameter settings were as follows: BC-NLMS used $\alpha = 0.95$ and $\rho = \sigma_v^2 / \sigma_\eta^2$; BC-NSAF used $\alpha = 0.95$ and $\kappa = \sqrt{\sigma_v^2}$; and the PFNVE-based BC-NLMS used $\alpha = 0.95$, $N_w = 5$, $\epsilon = 0.0001$. The window length $N_w = 5$ was selected after comparing $N_w = 3, 5$, and 10; it provides a good balance between estimation stability and tracking responsiveness. Figures 1–6 present the simulation results for randomly generated 32-tap systems, except Figure 3, which involves 16- and 64-tap systems. Figure 1 shows results for input SNR of 10 dB, where Figure 1a,b represents output SNRs of 10 dB and 20 dB respectively. Without requiring noise power ratios or output noise power, the algorithm achieves comparable steady-state accuracy.

Figure 2 illustrates results for input SNR equal to 20 dB, with Figure 2a,b corresponding to output SNRs of 10 dB and 20 dB respectively. Consistent with Figure 1, the PFNVE-based BC-NLMS maintains performance comparable to traditional algorithms. Figures 1 and 2 show consistent performance across the tested SNR conditions.

To provide a quantitative comparison, Table 2 reports the steady-state NMSD values averaged over the last 500 iterations for each algorithm under the four SNR conditions of Figures 1 and 2. The PFNVE-based BC-NLMS achieves steady-state NMSD within approximately 0.5 dB of BC-NLMS and BC-NSAF in all cases, despite requiring no prior noise information. The small gap arises from the median filter, which introduces a slight downward bias in the variance estimate due to its nonlinear averaging property. This difference is within an acceptable range considering that the PFNVE operates without any prior noise statistics.

Table 2. Steady-state NMSD (dB) averaged over iterations 3500–4000 (32-tap dense system).

Algorithm	iSNR = 10, oSNR = 10	iSNR = 10, oSNR = 20	iSNR = 20, oSNR = 10	iSNR = 20, oSNR = 20
NLMS	−15.33	−17.21	−18.93	−26.13
BC-NLMS	−16.01	−18.59	−18.89	−26.27
BC-NSAF	−16.27	−18.99	−18.75	−26.28
PFNVE-based BC-NLMS	−16.26	−18.55	−18.46	−26.18

Figure 3 shows the results for 16-tap and 64-tap systems, both tested at 10 dB input and output SNR. The PFNVE-based BC-NLMS reaches a steady-state MSD level close to those of BC-NLMS and BC-NSAF in both cases, which indicates that the estimator works well regardless of the filter length and does not require re-tuning of the parameters.

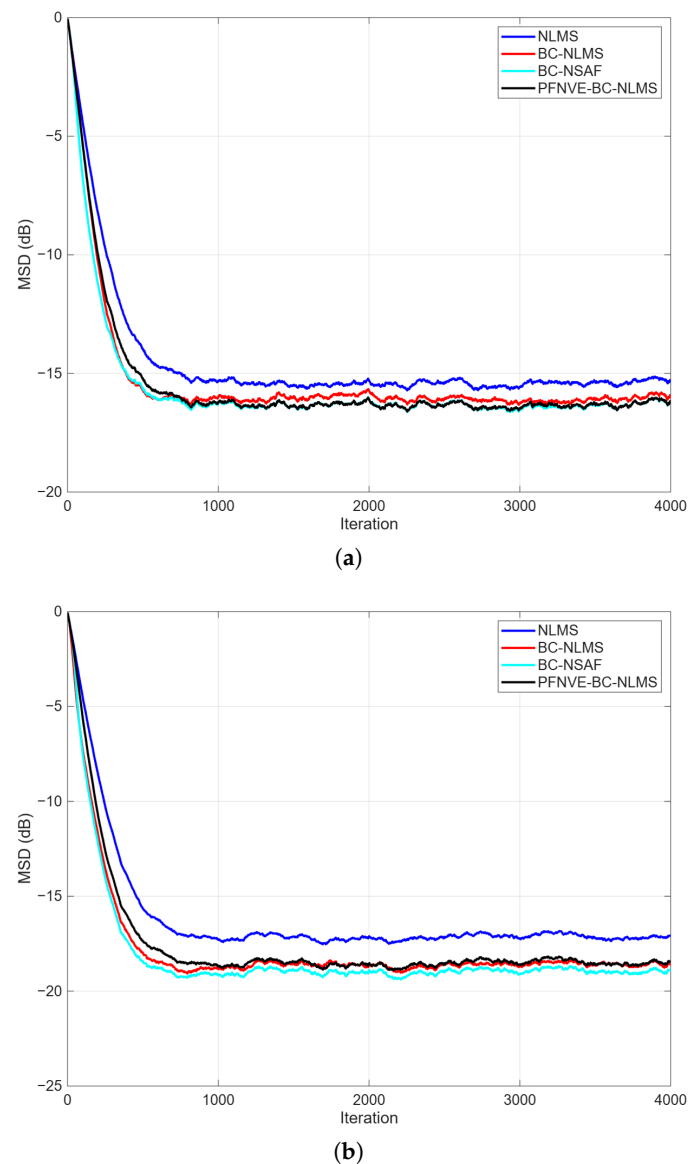


Figure 1. MSD performance of NLMS, BC-NLMS, BC-NSAF, and PFNVE-based BC-NLMS for a randomly generated dense system with 32 taps. Input SNR is 10 dB; (a) output SNR is 10 dB, (b) output SNR is 20 dB.

To assess tracking capability [37], the true system coefficients are multiplied by -1 at iteration 4000, creating an abrupt parameter change. The resulting learning curves are shown in Figure 4. We measure the recovery time as the number of iterations needed for the NMSD to fall back within 1 dB of its pre-change steady-state level. In this experiment, the PFNVE-based BC-NLMS recovers after roughly 1000 iterations, which is about the same time it took to converge initially. BC-NLMS, on the other hand, needs about 1500 iterations, and BC-NSAF takes more than 2500 iterations to reach a comparable level. This rapid readaptation is a direct consequence of the prior-free design. Since the estimator recalculates the noise variance from the current filter state at every iteration, it avoids the influence of stale parameters from the previous environment.

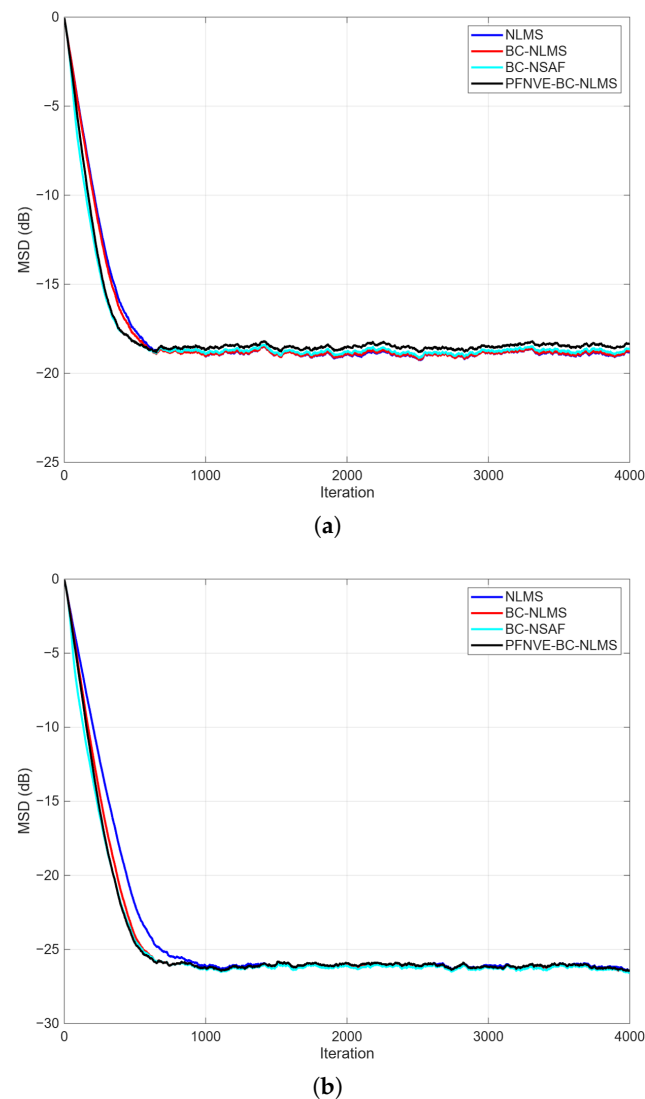


Figure 2. MSD performance comparison of NLMS, BC-NLMS, BC-NSAF, and PFNVE-based BC-NLMS for input SNR of 20 dB and 32-tap system; (a) output SNR is 10 dB, (b) output SNR is 20 dB.

To evaluate robustness under non-stationary noise conditions, the output noise variance was doubled at iteration 2000 while all other parameters remained unchanged. BC-NLMS and BC-NSAF were initialized with the exact prior information for the original noise environment. As shown in Figure 5, all three bias-compensated algorithms achieve similar steady-state MSD levels of approximately -16 dB, confirming that conventional methods perform well when given accurate prior knowledge. However, after the noise change, BC-NLMS degrades to approximately -13.5 dB because its fixed noise ratio parameter ρ no longer matches the current environment. BC-NSAF exhibits a slightly less severe degradation to about -14.5 dB, as the square-root relationship in its threshold τ partially mitigates the parameter mismatch. In contrast, the PFNVE-based BC-NLMS re-estimates the noise variance at every iteration and settles at approximately -14.8 dB. While this is comparable to BC-NSAF, it represents a clear improvement over BC-NLMS, and is achieved without relying on any prior noise parameters.

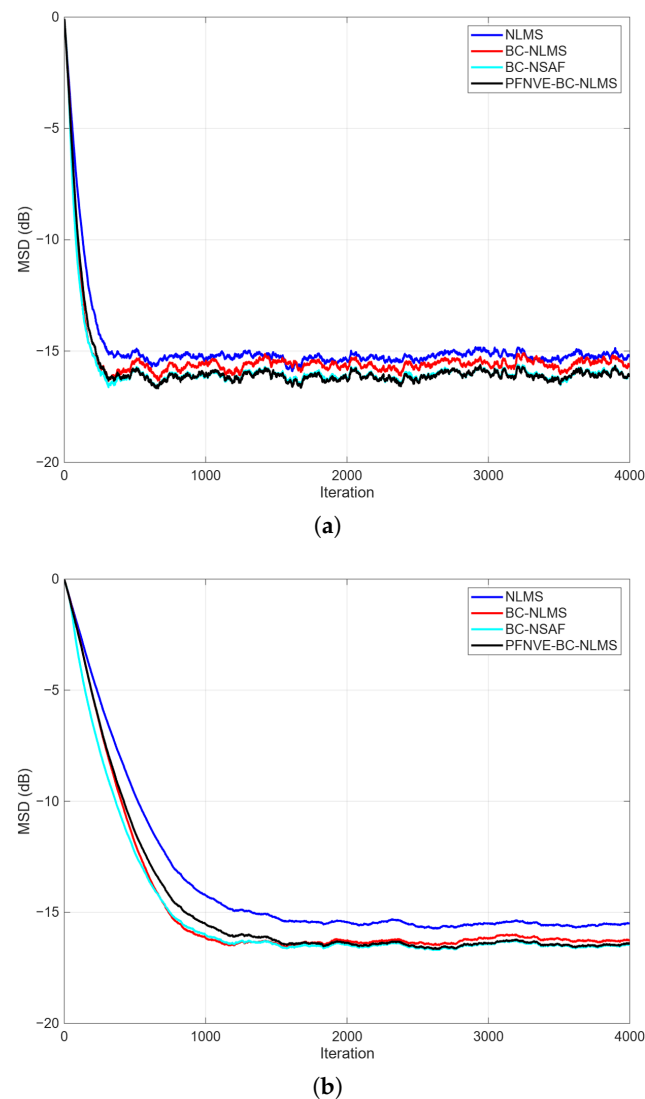


Figure 3. MSD performance of tested algorithms for varying tap lengths under 10 dB input and output SNR: (a) 16 taps, (b) 64 taps.

Figure 6 evaluates the tracking performance with a colored input signal generated by a first-order autoregressive model, $x(n) = 0.5x(n-1) + \zeta(n)$, where $\zeta(n)$ is a white Gaussian noise sequence. The remaining simulation conditions are identical to those in Figure 1a. Before the system change at iteration 4000, all three bias-compensated algorithms achieve similar steady-state MSD levels. However, following the sign reversal, the algorithms exhibit different tracking behaviors. The PFNVE-based BC-NLMS readapts and converges within approximately 1500 iterations. In contrast, BC-NLMS fails to return to its original steady-state level within the simulation period. BC-NSAF eventually recovers, but it requires nearly 4000 iterations. The poor tracking of BC-NLMS under colored input is expected, as its variance estimator in (4) assumes a white input. When the input is correlated, this assumption no longer holds and the ratio-based estimate becomes inaccurate, particularly after an abrupt system change where the weight-error vector is large. BC-NSAF is less affected since its shrinkage operates on the scalar error signal, but its recovery is still slower than the PFNVE-based BC-NLMS, which does not rely on any assumption about the input statistics.

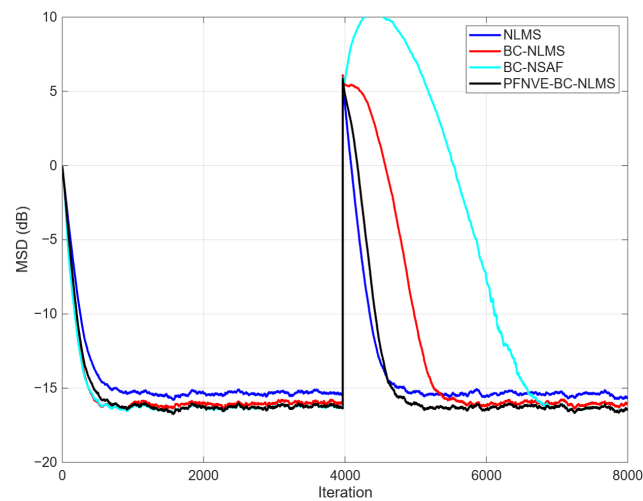


Figure 4. Tracking performance comparison for adaptive filters; system parameters reversed at iteration 4000.

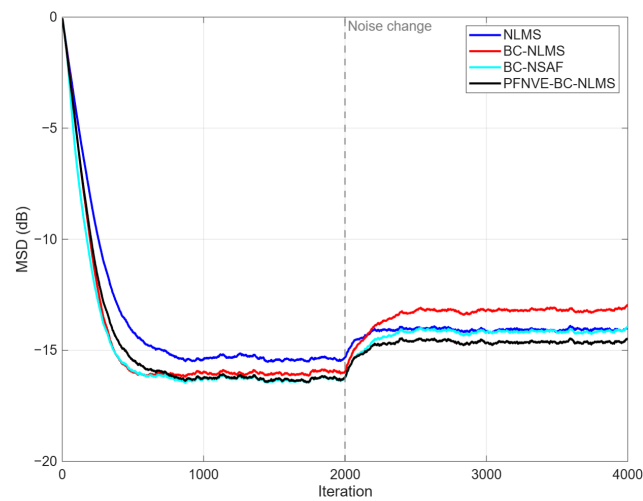


Figure 5. MSD performance under time-varying noise conditions; output noise variance doubled at iteration 2000.

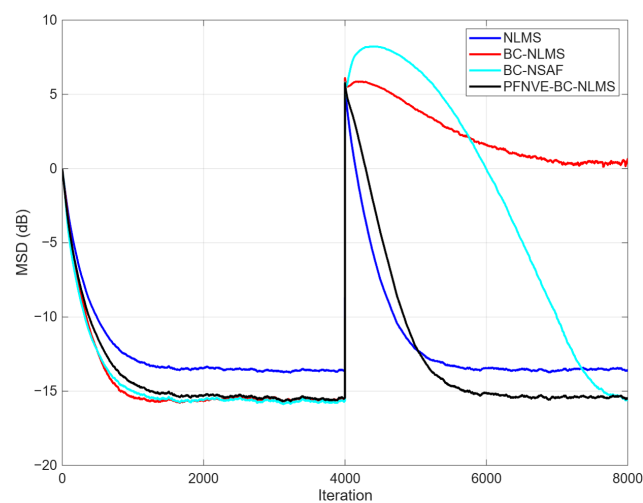


Figure 6. Tracking performance with colored AR(1) input ($a = 0.5$); system parameters reversed at iteration 4000.

4.2. Sparse System

Performance in sparse systems is evaluated through comparison with BC-MS-PNLMS. The unknown system has 64 taps, with sparsity levels of 2 and 48 considered. The sparsity level indicates the number of nonzero coefficients; a sparsity of 2 represents a highly sparse channel where only two taps carry signal energy, while a sparsity of 48 represents a weakly sparse system. Input and output SNRs were set to 10 dB and 20 dB, respectively. For a fair comparison, all algorithms used a step size of 0.4, $\delta = 5/M$, $\gamma = 0.1$, and $b = 1/128$. BC-MS-PNLMS parameters were set to $\alpha = 0.6$ and $\rho = \sigma_v^2 / \sigma_\eta^2$ [35], while the PFNVE used $\alpha = 0.95$ and $N_w = 5$.

The PFNVE is applied to the MS-PNLMS structure by replacing the ratio-based noise variance estimator in (16) with the orthogonality-based estimator in (21). No other modification to the gain matrix computation or the weight update is required. Figure 7 illustrates the MSD learning curves for sparsity levels of 2 (Figure 7a) and 48 (Figure 7b). In both cases, the PFNVE-based MS-PNLMS matches the steady-state accuracy and tracking speed of BC-MS-PNLMS. These results show that the prior-free estimator extends beyond the standard NLMS framework, maintaining robust performance across varying degrees of system sparsity without requiring prior noise information.

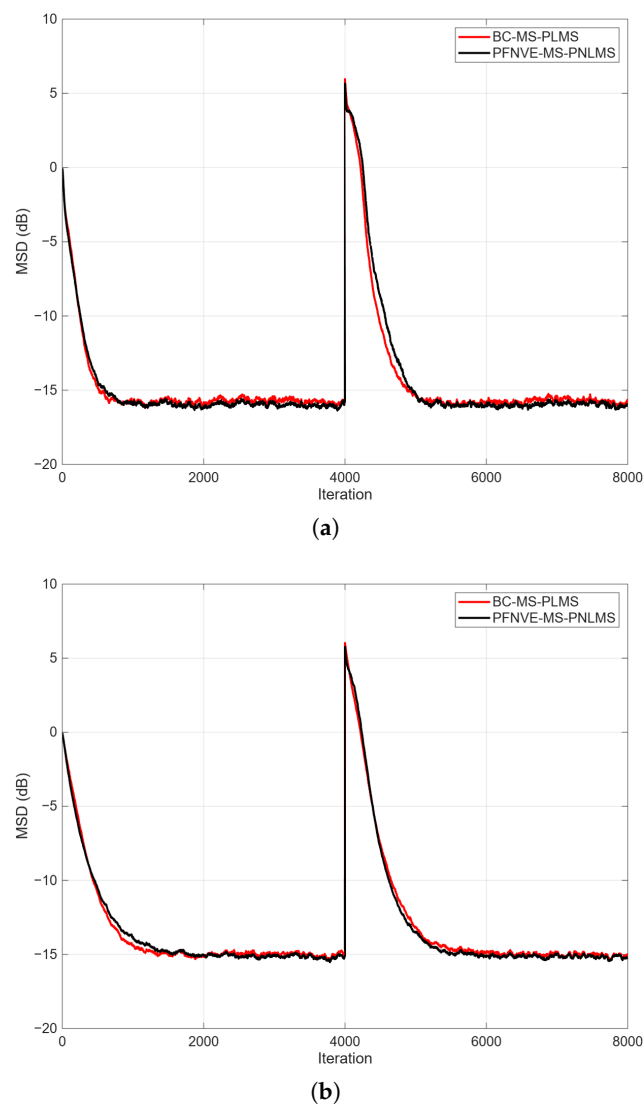


Figure 7. MSD performance comparison between BC-MS-PNLMS and PFNVE-based MS-PNLMS in sparse systems: (a) sparsity = 2, (b) sparsity = 48.

5. Conclusions

A bias-compensated adaptive filter has been presented in which the input noise variance estimator operates without any prior knowledge of the noise environment. The estimator is derived from the orthogonality between the weight-error vector and the current weight vector, a relation that holds near convergence, and its computational cost per iteration is comparable to that of existing methods. System identification experiments on both dense and sparse channels show that the algorithm achieves a steady-state mean-square deviation comparable to BC-NLMS and BC-NSAF, with faster recovery after abrupt system changes. Additional experiments with time-varying output noise and colored input show that the estimator can track changes in the noise statistics, whereas methods that rely on fixed prior parameters exhibit degradation. The estimation step reuses quantities that are already available in the weight update, so it can be incorporated into other LMS-family filters without modification; this has been verified here with the MS-PNLMS structure.

Several directions remain open for future work. On the theoretical side, extending the convergence analysis to colored input noise conditions is a natural next step, as the current derivation assumes white input noise. From a practical standpoint, an adaptive rule for the median filter window N_w could improve the balance between estimation stability and tracking speed. Extensions to complex-valued implementations and filtered-x variants such as FxNLMS are also worthwhile directions, particularly for communications and active noise control applications, where evaluation on real-world data would further establish the practical limits of the proposed method.

Funding: This research was supported by the regional innovation system & education (RISE)-(Regional Growth Innovation LAB) program through the Gyeongbuk RISE Center, funded by the Ministry of Education (MOE) and the Gyeongsangbuk-do, Republic of Korea (2026-rise-15-105).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: No new data were created or analyzed in this study.

Conflicts of Interest: The author declares no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

NLMS	Normalized Least Mean Square
C-NLMS	Consistent Normalized Least Mean Square
BC-NLMS	Bias-Compensated Normalized Least Mean Square
BC-NSAF	Bias-Compensated Normalized Subband Adaptive Filter
MS-PNLMS	Proportionate NLMS with Multi-Segment Function
BC-MS-PNLMS	Bias-Compensated MS-PNLMS
PFNVE	Prior-Free Noise Variance Estimator
PFNVE-BC-NLMS	PFNVE-based Bias-Compensated NLMS
PFNVE-MS-PNLMS	PFNVE-based Bias-Compensated MS-PNLMS
FxNLMS	Filtered-x Normalized Least Mean Square
MSD	Mean Square Deviation
NMSD	Normalized Mean Square Deviation
SNR	Signal-to-Noise Ratio
ANC	Active Noise Control
AR	Autoregressive

References

- Lee, K.A.; Gan, W.S.; Kuo, S.M. *Subband Adaptive Filtering: Theory and Implementation*; John Wiley & Sons: Hoboken, NJ, USA, 2009.
- Sayed, A.H. *Adaptive Filters*; John Wiley & Sons: Hoboken, NJ, USA, 2011.
- Zhang, F.; Wu, Y.; Wang, Y.; Li, X. Dynamic Matching of Reconstruction and Anti-Aliasing Filters in Adaptive Active Noise Control. *Appl. Sci.* **2024**, *14*, 4810. [\[CrossRef\]](#)
- Joel, S.; Yadav, S.K.; George, N.V. Enhanced bias-compensated NLMS for adaptive DOA estimation. *IEEE Sens. Lett.* **2024**, *8*, 6005004.
- Li, Y.; Zhu, Z.; Zhong, Z.; Wang, G. Fault Diagnosis Method of Motor Bearing Under Variable Load Condition Based on Parameter Optimization VMD-NLMS. *Appl. Sci.* **2025**, *15*, 2607. [\[CrossRef\]](#)
- Aali, S.R.; Besmi, M.R.; Kazemi, M.H. Smart VRP-NLMS algorithm for estimation of power system frequency. *COMPEL Int. J. Comput. Math. Electr. Electron. Eng.* **2019**, *38*, 362–381. [\[CrossRef\]](#)
- Das, S.; Puchalapalli, S.; Singh, B. Arctangent function-based normalized least mean squares algorithm for a battery supported wind energy conversion system. *IEEE J. Emerg. Sel. Top. Power Electron.* **2021**, *10*, 3907–3916. [\[CrossRef\]](#)
- Shi, W.; Li, Y.; Yin, J. Improved constraint NLMS algorithm for sparse adaptive array beamforming control applications. *Appl. Comput. Electromagn. Soc. J. (ACES)* **2019**, *34*, 419–423.
- So, H.C. LMS-based algorithm for unbiased FIR filtering with noisy measurements. *Electron. Lett.* **2001**, *37*, 1418–1420. [\[CrossRef\]](#)
- Zhang, Z.; Jia, L.; Tao, R.; Wang, Y. Blind adaptive identification and equalization using bias-compensated NLMS methods. *Sci. China Inf. Sci.* **2022**, *65*, 152302. [\[CrossRef\]](#)
- Widrow, B.; Stearns, S.D. *Adaptive Signal Processing*; Prentice-Hall: Hoboken, NJ, USA, 1985; Volume 3, pp. 362–367.
- Haykin, S.S. *Adaptive Filter Theory*; Pearson Education India: Chennai, India, 2008.
- Söderström, T. Errors-in-variables methods in system identification. *Automatica* **2007**, *43*, 939–958. [\[CrossRef\]](#)
- Ficiu, I.D.; Stanciu, C.L.; Paleologu, C.; Benesty, J. Low-complexity data-reuse RLS algorithm for stereophonic acoustic echo cancellation. *Appl. Sci.* **2023**, *13*, 2227. [\[CrossRef\]](#)
- Yu, H.; Zhang, H.; Xiang, J.; Yang, H. Neural Momentum-Enhanced LMS for Linear Acoustic Echo Cancellation. *IEEE Trans. Audio, Speech Lang. Process.* **2025**, *33*, 4574–4589. [\[CrossRef\]](#)
- Seidel, E.; Enzner, G.; Mowlae, P.; Fingscheidt, T. Neural Kalman Filters for Acoustic Echo Cancellation: Comparison of deep neural network-based extensions. *IEEE Signal Process. Mag.* **2025**, *41*, 24–38. [\[CrossRef\]](#)
- Shao, S.; Lv, T.; Huang, P. Adaptive Filtering for Channel Estimation in RIS-Assisted mmWave Systems. *Sensors* **2025**, *25*, 297. [\[CrossRef\]](#) [\[PubMed\]](#)
- Liu, Y.; Lei, Z. Review of advances in active impulsive noise control with focus on adaptive algorithms. *Appl. Sci.* **2024**, *14*, 1218. [\[CrossRef\]](#)
- Li, X.; Zheng, Z.; Shao, Z.; Han, Y. Active Impulsive Noise Control with Missing Input Data Based on FxImdMCC Algorithm. *Electronics* **2024**, *13*, 4319. [\[CrossRef\]](#)
- Han, R.; Wu, M.; Gong, C.; Jia, S.; Han, T.; Sun, H.; Yang, J. Combination of robust algorithm and head-tracking for a feedforward active headrest. *Appl. Sci.* **2019**, *9*, 1760. [\[CrossRef\]](#)
- Ying, L.; Wang, J.; Liu, Q.; Wang, D. Application study of adaptive tracking algorithm in active noise control system of transformer. *Appl. Sci.* **2019**, *9*, 2693. [\[CrossRef\]](#)
- Ma, W.; Zheng, D.; Li, Y.; Zhang, Z.; Chen, B. Bias-compensated normalized maximum correntropy criterion algorithm for system identification with noisy input. *Signal Process.* **2018**, *152*, 160–164. [\[CrossRef\]](#)
- Zhang, S.; Zhang, J.; Han, H. Robust variable step-size decorrelation normalized least-mean-square algorithm and its application to acoustic echo cancellation. *IEEE/ACM Trans. Audio Speech Lang. Process.* **2016**, *24*, 2368–2376. [\[CrossRef\]](#)
- Jo, S.; Kim, S.W. Consistent normalized least mean square filtering with noisy data matrix. *IEEE Trans. Signal Process.* **2005**, *53*, 2112–2123. [\[CrossRef\]](#)
- Jung, S.M.; Park, P. Stabilization of a bias-compensated normalized least-mean-square algorithm for noisy inputs. *IEEE Trans. Signal Process.* **2017**, *65*, 2949–2961. [\[CrossRef\]](#)
- Zheng, Z.; Zhao, H. Bias-compensated normalized subband adaptive filter algorithm. *IEEE Signal Process. Lett.* **2016**, *23*, 809–813. [\[CrossRef\]](#)
- Zhao, H.; Liu, D.; He, X. Bias-compensated sign subband adaptive filter algorithm with individual weighting factors for input noise. *IEEE Trans. Circuits Syst. II Express Briefs* **2021**, *69*, 1872–1876. [\[CrossRef\]](#)
- Song, F.; Huang, F.; Zhang, S.; Chen, H.; Wen, P. Robust Bias-Compensated Subband Adaptive Filtering. *IEEE Trans. Circuits Syst. II Express Briefs* **2023**, *70*, 3679–3683. [\[CrossRef\]](#)
- Ni, J.; Gao, Y.; Chen, X.; Chen, J. Bias-compensated sign algorithm for noisy inputs and its step-size optimization. *IEEE Trans. Signal Process.* **2021**, *69*, 2330–2342. [\[CrossRef\]](#)
- Zhao, H.; Liu, D.; Lv, S. Robust Maximum Correntropy Criterion Subband Adaptive Filter Algorithm for Impulsive Noise and Noisy Input. *IEEE Trans. Circuits Syst. II Express Briefs* **2022**, *69*, 604–608. [\[CrossRef\]](#)

31. Liu, D.; Zhao, H. Bias-compensated subband adaptive filter algorithm based on maximum correntropy criterion. In *2021 IEEE 16th Conference on Industrial Electronics and Applications (ICIEA)*; IEEE: New York, NY, USA, 2021; pp. 1202–1206.
32. Park, C.; Ryu, S.H.; Park, P. A variable step size bias-compensated affine projection algorithm with noisy inputs. *J. Frankl. Inst.* **2025**, *362*, 107792. [[CrossRef](#)]
33. Gu, Y.; Song, Q.; Ni, J. Bias-compensated constrained adaptive filtering algorithm for noisy inputs. *Signal Process.* **2026**, *238*, 110174. [[CrossRef](#)]
34. Shi, D.; Lam, B.; Ooi, K.; Shen, X.; Gan, W.S. Selective fixed-filter active noise control based on convolutional neural network. *Signal Process.* **2022**, *190*, 108317. [[CrossRef](#)]
35. Jin, Z.; Yang, Z.; Li, Q.; Ma, L. Bias-Compensated PNLMS Algorithm with Multi-Segment Function For Noisy Input. *IEEE Access* **2025**, *13*, 43741–43748. [[CrossRef](#)]
36. Arce, G.R. *Nonlinear Signal Processing: A Statistical Approach*; John Wiley & Sons: Hoboken, NJ, USA, 2004.
37. Benesty, J.; Rey, H.; Vega, L.; Tressens, S. A Nonparametric VSS NLMS Algorithm. *IEEE Signal Process. Lett.* **2006**, *13*, 581–584. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.